

On radiative $np \rightarrow 1s + \gamma$ transitions, induced by strong low-energy interactions, in kaonic atoms

A. N. Ivanov^{*†}, M. Cargnelli[‡], M. Faber[§], H. Fuhrmann[¶],
V. A. Ivanova^{||}, J. Marton^{**}, N. I. Troitskaya^{††}, J. Zmeskal^{‡‡}

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*Atominstitut der Österreichischen Universitäten, Arbeitsbereich Kernphysik und
Nukleare Astrophysik, Technische Universität Wien,
Wiedner Hauptstr. 8-10, A-1040 Wien, Österreich*

and

*Stefan Meyer Institut für subatomare Physik der Österreichische Akademie der
Wissenschaften,
Boltzmannngasse 3, A-1090, Wien, Österreich*

Abstract

We calculate the rates of the radiative transitions $np \rightarrow 1s + \gamma$ in kaonic hydrogen and kaonic deuterium, induced by strong low-energy interactions and enhanced by Coulomb interactions. The obtained results should be taken into account for the theoretical analysis of the experimental data on the X -ray spectra and yields in kaonic atoms.

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^{*}E-mail: ivanov@kph.tuwien.ac.at, Tel.: +43-1-58801-14261, Fax: +43-1-58801-14299

[†]Permanent Address: State Polytechnic University, Department of Nuclear Physics, 195251 St. Petersburg, Russian Federation

[‡]E-mail: michael.cargnelli@oeaw.ac.at

[§]E-mail: faber@kph.tuwien.ac.at, Tel.: +43-1-58801-14261, Fax: +43-1-58801-14299

[¶]E-mail: hermann.fuhrmann@oeaw.ac.at

^{||}E-mail: viola@kph.tuwien.ac.at, State Polytechnic University, Department of Nuclear Physics, 195251 St. Petersburg, Russian Federation

^{**}E-mail: johann.marton@oeaw.ac.at

^{††}State Polytechnic University, Department of Nuclear Physics, 195251 St. Petersburg, Russian Federation

^{‡‡}E-mail: johann.zmeskal@oeaw.ac.at

1 Introduction

The X -ray spectra and yields [1]–[7], produced by the atomic transitions $np \rightarrow 1s$ in kaonic hydrogen, where n is the *principle quantum number* of the energy levels, are the main experimental tool for the measurement of the energy level displacement of the ground state of kaonic hydrogen, caused by strong low-energy $\bar{K}N$ interactions [8]. It is known [4]–[7] that the X -ray yields related to the K_α , K_β and K_γ lines of kaonic hydrogen are very sensitive to the value of Γ_{2p} , the rate of hadronic decays of kaonic hydrogen from the $2p$ state. Usually Γ_{2p} is used as an input parameter in the theories of the atomic cascades [4]–[7].

Recently [9] we have calculated the rate Γ_{np} of hadronic decays of kaonic hydrogen from the np state. For the $2p$ state we have obtained: $\Gamma_{2p} = 2 \text{ meV} = 3.0 \times 10^{12} \text{ sec}^{-1}$. This agrees well with the assumption by Koike, Harada, and Akaishi [4]. In order to reconcile the experimental data on the K_α , K_β and K_γ lines with the theoretical analysis they assumed that $\Gamma_{2p} > 1 \text{ meV} = 1.5 \times 10^{12} \text{ sec}^{-1}$.

In this paper we continue the analysis of the influence of strong low-energy interactions on the transitions from the np state in kaonic hydrogen, which we have started in [9]. We investigate the radiative transitions $np \rightarrow 1s + \gamma$, induced by strong low-energy interactions and enhanced by the Coulomb interaction of the K^-p pair, in kaonic hydrogen and extend the results on kaonic deuterium.

The paper is organized as follows. In Section 2 we calculate the rates of the radiative transitions $np \rightarrow 1s + \gamma$ in kaonic hydrogen, induced by strong low-energy interactions and enhanced by the Coulomb interaction. The Coulomb interaction is taken into account in the form of the explicit non-relativistic Coulomb wave functions of the relative motion of the K^-p pairs and the explicit non-relativistic Coulomb Green functions for the calculation of the amplitude of the kaon-proton *Bremsstrahlung* $K^- + p \rightarrow K^- + p + \gamma$, defining the rate of the radiative atomic transition $(K^-p)_{np} \rightarrow (K^-p)_{1s} + \gamma$ in our approach. In Section 3 we modify the rates of the transitions $np \rightarrow 1s + \gamma$, calculated in Section 2 for kaonic hydrogen, for kaonic deuterium. In the Conclusion we discuss the obtained results.

2 $np \rightarrow 1s + \gamma$ transitions in kaonic hydrogen

The rate of the radiative transition $(K^-p)_{np} \rightarrow (K^-p)_{1s} + \gamma$ can be defined by [10]

$$\Gamma((K^-p)_{np} \rightarrow (K^-p)_{1s} \gamma) = \frac{1}{8\pi} \frac{\omega}{(m_K + m_N)^2} \overline{|M((K^-p)_{np} \rightarrow (K^-p)_{1s} \gamma)|^2}, \quad (2.1)$$

where $\omega = E_{np} - E_{1s} = \alpha^2 \mu (n^2 - 1)/2n^2$ is the photon energy, $\alpha = 1/137.036$ is the fine-structure constant, $\mu = m_K m_N / (m_K + m_N) = 324 \text{ MeV}$ is the reduced mass of the K^-p pair calculated for $m_K = 494 \text{ MeV}$ and $m_N = 940 \text{ MeV}$. The amplitude $M((K^-p)_{np} \rightarrow (K^-p)_{1s} \gamma)$ is given by [10]

$$M((K^-p)_{np} \rightarrow (K^-p)_{1s} \gamma) = \frac{1}{2\mu} \int \frac{d^3q}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \Phi_{100}^*(\vec{k}) \Phi_{n1m}(\vec{q}) M(K^-p \rightarrow K^-p \gamma), \quad (2.2)$$

where $\Phi_{100}(\vec{k}) = \Phi_{1s}(k)$ and $\Phi_{n1m}(\vec{q}) = -i\sqrt{4\pi} \Phi_{np}(q) Y_{1m}(\vartheta_{\vec{q}}, \varphi_{\vec{q}})$ are the wave functions of kaonic hydrogen in the ground $1s$ and the np state in the momentum representation

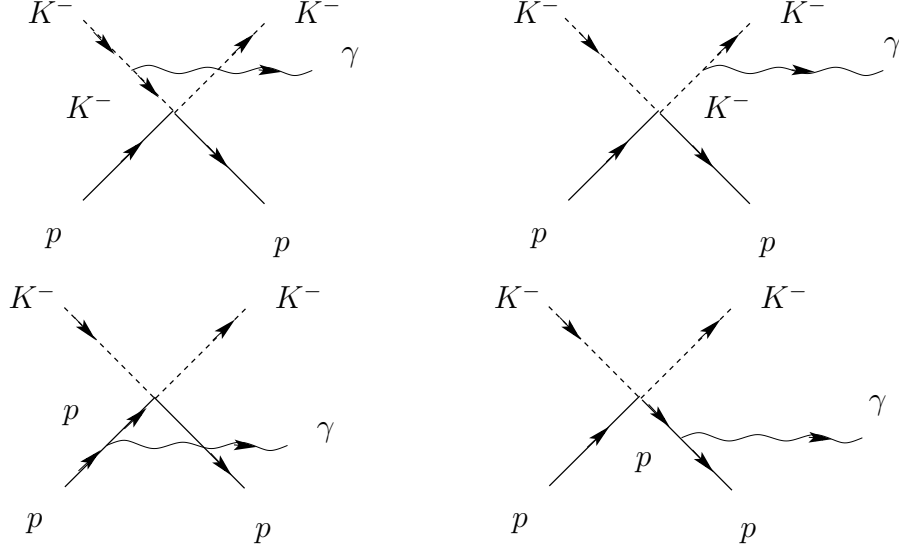


Figure 1: Feynman diagrams for the radiative transitions $(K^-p)_{np} \rightarrow (K^-p)_{1s} + \gamma$ caused by strong low-energy interactions

[11]; $Y_{1m}(\vartheta_{\vec{q}}, \varphi_{\vec{q}})$ is a spherical harmonic and $\Phi_{1s}(k)$ and $\Phi_{np}(q)$ are radial wave functions of kaonic hydrogen in the momentum representation [11] (see also [9, 12]). Then, $M(K^-p \rightarrow K^-p\gamma)$ is the amplitude of the kaon-proton *Bremsstrahlung* $K^- + p \rightarrow K^- + p + \gamma$.

The amplitude $M(K^-p \rightarrow K^-p\gamma)$ is defined by the Feynman diagrams depicted in Fig.1. For the calculation of this amplitude we use the following effective Lagrangian, describing strong low-energy and electromagnetic interactions of the K^-p pairs:

$$\begin{aligned} \mathcal{L}_{\text{eff}}(x) = & \partial_\mu K^{-\dagger}(x) \partial^\mu K^-(x) - m_K^2 K^{-\dagger}(x) K^-(x) + \bar{p}(x) (i\gamma^\mu \partial_\mu - m_N) p(x) \\ & + ie (K^{-\dagger}(x) \partial_\mu K^-(x) - \partial_\mu K^{-\dagger}(x) K^-(x)) A^\mu(x) - e \bar{p}(x) \gamma_\mu p(x) A^\mu(x) \\ & + 4\pi \left(1 + \frac{m_K}{m_N}\right) a_0^{K^-p} \bar{p}(x) p(x) K^{-\dagger}(x) K^-(x) + \dots, \end{aligned} \quad (2.3)$$

where e is the electric charge of the proton such as $e^2 = 4\pi\alpha$, $a_0^{K^-p}$ is the S-wave scattering length of K^-p scattering [10, 13], $A^\mu(x)$ is the vector potential of the quantized electromagnetic field. The ellipses denote interactions of order of $O(e^2)$, which we omit. In the effective Lagrangian Eq.(2.3) strong low-energy interactions of the K^-p pair are described by $\mathcal{L}_{KKpp}(x) = 4\pi(1 + m_K/m_N) a_0^{K^-p} \bar{p}(x) p(x) K^{-\dagger}(x) K^-(x)$.

In the non-relativistic limit the amplitude of the kaon-proton *Bremsstrahlung* $K^- + p \rightarrow K^- + p + \gamma$ in Fig.1 reads

$$M(K^-p \rightarrow K^-p\gamma) = 8\pi (m_K + m_N) a_0^{K^-p} \frac{ie}{2\mu\omega} \vec{e}^*(\vec{p}, \lambda) \cdot (\vec{k} + \vec{q}). \quad (2.4)$$

For the calculation of the amplitude of the atomic transition $(K^-p)_{np} \rightarrow (K^-p)_{1s} + \gamma$ the amplitude of the kaon-proton *Bremsstrahlung* Eq.(2.4) should be weighted with the wave functions of kaonic hydrogen in the ground and excited np states Eq.(2.2). Since the wave function of the ground state is spherical symmetric, the integration over \vec{k} leads to the vanishing of the term proportional to $\vec{k} \cdot \vec{e}^*(\vec{p}, \lambda)$. Therefore, below we omit it.

The appearance of the photon energy ω in the denominator of the amplitude of the kaon–proton *Bremsstrahlung* $K^- + p \rightarrow K^- + p + \gamma$ is due to the fact that the virtual K^- –meson and the proton are practically on–mass shell.

Our approach to the calculation of the amplitude of the kaon–proton *Bremsstrahlung* $K^- + p \rightarrow K^- + p + \gamma$ is similar to that which has been used in [13] for the derivation of the Ericson–Weise formula for the S–wave scattering amplitude of K^-d scattering [14] and runs parallel the non–relativistic Effective Field Theory (the EFT) approach, based on Chiral Perturbation Theory (ChPT) by Gasser and Leutwyler [15], which has been applied by Meißner *et al.* [16] to the systematic calculation of QCD isospin–breaking and electromagnetic corrections to the energy level displacement of the ns state of kaonic hydrogen. In [9] we have analysed the quantitative agreement of the results on the energy level displacement of the ns state of kaonic hydrogen obtained by Meißner’s group [16], our group [10, 13] and the experimental data by the DEAR Collaboration [8].

Due to the Coulomb interaction of the K^-p pair in initial and final states [17]–[19] the amplitude of the kaon–proton *Bremsstrahlung* Eq.(2.4) changes as follows

$$\begin{aligned} M(K^-p \rightarrow K^-p\gamma) &= 8\pi (m_K + m_N) a_0^{K^-p} \frac{ie}{2\mu\omega} \vec{e}^*(\vec{p}, \lambda) \cdot \vec{q} \\ &\times e^{\pi/2ka_B} \Gamma(1 - i/ka_B) e^{\pi/2qa_B} \Gamma(2 - i/qa_B), \end{aligned} \quad (2.5)$$

where $a_B = 1/\alpha\mu$ is the Bohr radius and the Γ –functions are defined by [20]

$$\begin{aligned} e^{\pi z/2} \Gamma(1 - iz) &= \sqrt{\frac{2\pi z}{1 - e^{-2\pi z}}} \exp \left\{ i \left[\gamma z + \sum_{k=1}^{\infty} \left(\frac{z}{k} - \arctan \left(\frac{z}{k} \right) \right) \right] \right\}, \\ e^{\pi z/2} \Gamma(2 - iz) &= (1 - iz) e^{\pi z/2} \Gamma(1 - iz) \end{aligned} \quad (2.6)$$

with Euler’s constant $\gamma = 0.57721 \dots$ [20].

For the calculation of the amplitude Eq.(2.5), taking into account the Coulomb interaction of the K^-p pair in the initial and final state, we have used the potential model approach. In this approach we describe strong low–energy interactions by the effective zero–range potential

$$V(\vec{r}) = -\frac{2\pi}{\mu} a_0^{K^-p} \delta^{(3)}(\vec{r}), \quad (2.7)$$

which is equivalent to the effective local strong low–energy $KKpp$ –interaction $\mathcal{L}_{KKpp}(x) = 4\pi (1 + m_K/m_N) a_0^{K^-p} \bar{p}(x)p(x)K^{-\dagger}(x)K^-(x)$ in Eq.(2.3).

Including the Coulomb interaction the term $i\vec{e}^*(\vec{p}, \lambda) \cdot \vec{q}$ in Eq.(2.4) becomes replaced by

$$i\vec{e}^*(\vec{p}, \lambda) \cdot \vec{q} \rightarrow \vec{e}^*(\vec{p}, \lambda) \cdot \int d^3x \delta^{(3)}(\vec{r}) \nabla \psi_{K^-p}^C(\vec{q}, \vec{r}) = i\vec{e}^*(\vec{p}, \lambda) \cdot \vec{q} e^{\pi/2qa_B} \Gamma(2 - i/qa_B), \quad (2.8)$$

where $\psi_{K^-p}^C(\vec{q}, \vec{r})$ is the exact non–relativistic Coulomb wave function of the relative motion of the K^-p pair in the incoming scattering state with relative momentum \vec{q} . It is given by [19]

$$\psi_{K^-p}^C(\vec{q}, \vec{r}) = e^{\pi/2qa_B} \Gamma(1 - i/qa_B) e^{i\vec{q} \cdot \vec{r}} F(i/qa_B, 1, iqr - i\vec{q} \cdot \vec{r}). \quad (2.9)$$

Here $F(i/qa_B, 1, i\vec{q} \cdot \vec{r})$ is the confluent function [19, 20].

The factor $e^{\pi/2ka_B} \Gamma(1 - i/ka_B)$ in Eq.(2.5) is the rest of the asymptotic of the non-relativistic Coulomb Green function $G_{K^-p}^C(\vec{r}, 0; k^2)$ [18]

$$G_{K^-p}^C(\vec{r}, 0; k^2) = -\frac{1}{4\pi r} \Gamma(1 - i/ka_B) W_{i/ka_B, 1/2}(-2ikr) \quad (2.10)$$

of the relative motion of the K^-p pair at $r \rightarrow \infty$, where $W_{i/ka_B, 1/2}(-2ikr)$ is the Whittaker function [20], describing the outgoing spherical wave distorted by the Coulomb interaction [18].

For the amplitude of the transition $(K^-p)_{np} \rightarrow (K^-p)_{1s} + \gamma$ we get

$$\begin{aligned} M((K^-p)_{np} \rightarrow (K^-p)_{1s} \gamma) &= 8\pi (m_K + m_N) a_0^{K^-p} \frac{ie}{4\mu^2\omega} \vec{e}^*(\vec{p}, \lambda) \cdot \\ &\int \frac{d^3k}{(2\pi)^3} \Phi_{100}^*(\vec{k}) e^{\pi/2ka_B} \Gamma(1 - i/ka_B) \int \frac{d^3q}{(2\pi)^3} e^{\pi/2qa_B} \Gamma(2 - i/qa_B) \vec{q} \Phi_{n1m}(\vec{q}). \end{aligned} \quad (2.11)$$

The main contributions to the integrals over \vec{k} and \vec{q} come from the regions $k \geq 1/a_B$ and $q > 1/na_B$ [14]. These momenta are of order of $O(\alpha)$. Therefore, the contribution of the photon momentum can be dropped, since it is of order of $O(\alpha^2)$.

For the integration over \vec{q} we define the vector \vec{q} in the spherical basis [13]

$$\vec{q} = q \sqrt{\frac{4\pi}{3}} \sum_{M=0, \pm 1} Y_{1M}^*(\vartheta_{\vec{q}}, \varphi_{\vec{q}}) \vec{e}_M, \quad (2.12)$$

where $\vec{e}_{\pm 1} = \mp(\vec{e}_x \pm i\vec{e}_y)/\sqrt{2}$ and $\vec{e}_0 = \vec{e}_z$ are spherical unit vectors expended into Cartesian unit vectors \vec{e}_x , \vec{e}_y and \vec{e}_z . Using Eq.(2.12) for the integral over \vec{q} we get [12]

$$\int \frac{d^3q}{(2\pi)^3} e^{\pi/2qa_B} \Gamma(2 - i/qa_B) \vec{q} \Phi_{n1m}(\vec{q}) = -i \frac{\vec{e}_m}{\sqrt{3}} \int \frac{d^3q}{(2\pi)^3} e^{\pi/2qa_B} \Gamma(2 - i/qa_B) q \Phi_{np}(q). \quad (2.13)$$

For the integration over k and q we use the wave functions $\Phi_{100}(\vec{k}) = \Phi_{1s}(k)$ and $\Phi_{np}(q)$ given by [11]

$$\Phi_{1s}(k) = \frac{8\sqrt{\pi a_B^3}}{(1 + k^2 a_B^2)^2}, \quad \Phi_{np}(q) \sqrt{\frac{\pi n^3 a_B^3}{n^2 - 1}} \frac{32 n q a_B}{(1 + n^2 q^2 a_B^2)^3} C_{n-2}^2\left(\frac{n^2 q^2 a_B^2 - 1}{n^2 q^2 a_B^2 + 1}\right), \quad (2.14)$$

where $C_{n-2}^2(z)$ is the Gegenbauer polynomial [11, 20].

Using [21] and taking into account the results obtained in [12] we make the integration over momenta k and q :

$$\begin{aligned} \int \frac{d^3k}{(2\pi)^3} \Phi_{1s}^*(k) e^{\pi/2ka_B} \Gamma(1 - i/ka_B) &\simeq \xi_{1s} \sqrt{\frac{1}{\pi a_B^3}} e^{i\varphi_{1s}}, \\ \int \frac{d^3q}{(2\pi)^3} e^{\pi/2qa_B} \Gamma(2 - i/qa_B) q \Phi_{np}(q) &\simeq \xi_{np} \sqrt{\frac{1}{\pi a_B^5} \frac{n^2 - 1}{n^5}} e^{i\varphi_{np}}, \end{aligned} \quad (2.15)$$

where $\xi_{1s} = 1.91$ and $\xi_{2p} = 3.52, \xi_{3p} = 2.22, \xi_{4p} = 2.85, \dots$ and φ_{1s} and φ_{np} are real phases, which do not contribute to the rate of the transition $(K^-p)_{np} \rightarrow (K^-p)_{1s} + \gamma$.

The amplitude of the transition $(K^-p)_{np} \rightarrow (K^-p)_{1s} + \gamma$ reads

$$M((K^-p)_{np} \rightarrow (K^-p)_{1s} \gamma) = 8\pi (m_K + m_N) \times i a_0^{K^-p} \frac{\mu^2}{\omega} \sqrt{\frac{\alpha^9}{4\pi}} \vec{e}^*(\vec{p}, \lambda) \cdot \frac{\vec{e}_m}{\sqrt{3}} \sqrt{\frac{n^2 - 1}{n^5}} \xi_{1s} \xi_{np} e^{i(\varphi_{1s} + \varphi_{np})}. \quad (2.16)$$

The rate of the transition $(K^-p)_{np} \rightarrow (K^-p)_{1s} + \gamma$ is equal to

$$\Gamma((K^-p)_{np} \rightarrow (K^-p)_{1s} \gamma) = \frac{8}{n^3} \frac{\xi_{np}^2}{\xi_{2p}^2} \Gamma((K^-p)_{2p} \rightarrow (K^-p)_{1s} \gamma), \quad (2.17)$$

where the rate $\Gamma((K^-p)_{2p} \rightarrow (K^-p)_{1s} \gamma)$ is defined by

$$\Gamma((K^-p)_{2p} \rightarrow (K^-p)_{1s} \gamma) = \frac{\xi_{1s}^2 \xi_{2p}^2}{9} \alpha^7 \mu^3 |a_0^{K^-p}|^2. \quad (2.18)$$

For the subsequent analysis of the rate of the transition $(K^-p)_{np} \rightarrow (K^-p)_{1s} + \gamma$ it is convenient to represent $|a_0^{K^-p}|^2$ in terms of the energy level displacement of the ground state of kaonic hydrogen

$$|a_0^{K^-p}|^2 = \frac{1}{4\alpha^6 \mu^4} \left(\epsilon_{1s}^2 + \frac{1}{4} \Gamma_{1s}^2 \right). \quad (2.19)$$

This is the model-independent DGBTT (Deser, Goldberger, Baumann, Thirring [22] and Trueman [23]) formula. Substituting Eq.(2.19) into Eq.(2.18) we get

$$\Gamma((K^-p)_{2p} \rightarrow (K^-p)_{1s} \gamma) = \frac{\xi_{1s}^2 \xi_{2p}^2}{36} \frac{\alpha}{\mu} \left(\epsilon_{1s}^2 + \frac{1}{4} \Gamma_{1s}^2 \right). \quad (2.20)$$

Using the numerical values of the parameters ξ_{1s} and ξ_{2p} we obtain

$$\Gamma((K^-p)_{2p} \rightarrow (K^-p)_{1s} \gamma) = 4.3 \times 10^4 \left(\epsilon_{1s}^2 + \frac{1}{4} \Gamma_{1s}^2 \right) \text{sec}^{-1}, \quad (2.21)$$

where ϵ_{1s} and Γ_{1s} are measured in eV.

The recent theoretical value of the energy level displacement of the ground state of kaonic hydrogen reads [10]

$$- \epsilon_{1s} + i \frac{\Gamma_{1s}}{2} = (-203 \pm 15) + i (113 \pm 14) \text{eV}. \quad (2.22)$$

Inserting Eq.(2.22) into Eq.(2.21) for the rate of the transition $\Gamma((K^-p)_{2p} \rightarrow (K^-p)_{1s} \gamma)$ we get

$$\Gamma((K^-p)_{2p} \rightarrow (K^-p)_{1s} \gamma) = (2.3 \pm 0.3) \times 10^9 \text{sec}^{-1} \quad (2.23)$$

According to Eq.(2.17), the rate of the transition $(K^-p)_{3p} \rightarrow (K^-p)_{1s} + \gamma$ is equal to

$$\Gamma((K^-p)_{3p} \rightarrow (K^-p)_{1s} \gamma) = (2.7 \pm 0.4) \times 10^8 \text{sec}^{-1}. \quad (2.24)$$

These rates should be compared with the rates of the pure electric dipole transitions $2p \rightarrow 1s + \gamma$ and $3p \rightarrow 1s + \gamma$ at the neglect of strong interactions.

Using the results obtained by Bethe and Salpeter [11] and adjusting them to kaonic hydrogen we get $\Gamma_{2p \rightarrow 1s} = 4.0 \times 10^{11} \text{ sec}^{-1}$ and $\Gamma_{3p \rightarrow 1s} = 1.0 \times 10^{11} \text{ sec}^{-1}$, respectively. Hence, the rates of the transitions $(K^-p)_{2p} \rightarrow (K^-p)_{1s} + \gamma$ and $(K^-p)_{3p} \rightarrow (K^-p)_{1s} + \gamma$, given by Eqs.(2.23) and (2.24) and induced by strong low-energy interactions, make up about 0.6 % and 0.3 % of the rates of the pure electric dipole transitions $2p \rightarrow 1s + \gamma$ and $3p \rightarrow 1s + \gamma$, respectively.

For the experimental values of the shift and width of the energy level of the ground state of kaonic hydrogen, measured by Iwasaki *et al.* (the KEK Collaboration) [24] : $(\epsilon_{1s}, \Gamma_{1s}) = (-323 \pm 64, 407 \pm 230) \text{ eV}$, the rates of the transitions $(K^-p)_{2p} \rightarrow (K^-p)_{1s} + \gamma$ and $(K^-p)_{3p} \rightarrow (K^-p)_{1s} + \gamma$, given by Eqs.(2.23) and (2.24), become increased by a factor of three.

3 $np \rightarrow 1s + \gamma$ transitions in kaonic deuterium

The formula (2.20) can be easily extended to radiative transitions in kaonic deuterium $(K^-d)_{np} \rightarrow (K^-d)_{1s} + \gamma$, induced by strong low-energy interactions. This reads

$$\Gamma((K^-d)_{2p} \rightarrow (K^-d)_{1s} \gamma) = \frac{\xi_{1s}^2 \xi_{2p}^2}{36} \frac{\alpha}{\mu} \left(\epsilon_{1s}^2 + \frac{1}{4} \Gamma_{1s}^2 \right) = 3.6 \times 10^4 \left(\epsilon_{1s}^2 + \frac{1}{4} \Gamma_{1s}^2 \right) \text{ sec}^{-1}, \quad (3.1)$$

where $\mu = m_K m_d / (m_K + m_d) = 391 \text{ MeV}$ is the reduced mass of the K^-d pair and $m_d = 1876 \text{ MeV}$ is the deuteron mass.

Recently the energy level displacement of the ground state of kaonic deuterium has been estimated in [13]:

$$- \epsilon_{1s} + i \frac{\Gamma_{1s}}{2} = (-325 \pm 60) + i (315 \pm 50) \text{ eV}. \quad (3.2)$$

According to Eq.(3.2), the rates of the radiative transitions $(K^-d)_{2p} \rightarrow (K^-d)_{1s} + \gamma$ and $(K^-d)_{3p} \rightarrow (K^-d)_{1s} + \gamma$ are equal to

$$\begin{aligned} \Gamma((K^-d)_{2p} \rightarrow (K^-d)_{1s} \gamma) &= (7.4 \pm 1.8) \times 10^9 \text{ sec}^{-1}, \\ \Gamma((K^-d)_{3p} \rightarrow (K^-d)_{1s} \gamma) &= (8.7 \pm 2.1) \times 10^8 \text{ sec}^{-1}. \end{aligned} \quad (3.3)$$

These rates should be compared with the rates $2p \rightarrow 1s + \gamma$ and $3p \rightarrow 1s + \gamma$ of the pure electric dipole transitions. Using the results obtained by Bethe and Salpeter [11] and adjusting them to kaonic deuterium we get $\Gamma_{2p \rightarrow 1s} = 4.8 \times 10^{11} \text{ sec}^{-1}$ and $\Gamma_{3p \rightarrow 1s} = 1.2 \times 10^{11} \text{ sec}^{-1}$, respectively.

Thus, the rates of the radiative transitions $(K^-d)_{2p} \rightarrow (K^-d)_{1s} + \gamma$ and $(K^-d)_{3p} \rightarrow (K^-d)_{1s} + \gamma$, induced by strong low-energy interactions, make up about 1.5% and 0.7 % of the rates of the pure electric dipole transitions $2p \rightarrow 1s + \gamma$ and $3p \rightarrow 1s + \gamma$, respectively.

For the value of the shift and width of the energy level of the ground state of kaonic deuterium, predicted by Barrett and Deloff [25]: $(\epsilon_{1s}^{(d)}, \Gamma_{1s}^{(d)}) = (-693, 880) \text{ eV}$, the rates of the transitions $(K^-d)_{2p} \rightarrow (K^-d)_{1s} + \gamma$ and $(K^-d)_{3p} \rightarrow (K^-d)_{1s} + \gamma$, given by Eq.(3.3), become increased by more than three times.

4 Conclusion

We have calculated the rates of the radiative transitions $np \rightarrow 1s + \gamma$ for kaonic hydrogen and kaonic deuterium, induced by strong low-energy interactions and enhanced by the attractive Coulomb interaction of the K^-p and K^-d pairs in the kaon-proton and kaon-deuteron *Bremsstrahlung*, $K^- + p \rightarrow K^- + p + \gamma$ and $K^- + d \rightarrow K^- + d + \gamma$. The neglect of the Coulomb interaction of the K^-p and K^-d pairs in these reactions corresponds to $\xi_{1s} = \xi_{np} = 1$ in Eq.(2.16). For the calculation of the amplitudes of the kaon-proton and kaon-deuteron *Bremsstrahlung* we have used an approach analogous to that of the EFT based on ChPT by Gasser and Leutwyler [15] and used in [13] for the derivation of the Ericson-Weise formula for the S-wave scattering length of K^-d scattering [14]. It agrees also well with the potential model approach.

We have found that for the $2p$ states of kaonic atoms the contributions of the rates of the transitions $(K^-p)_{2p} \rightarrow (K^-p)_{1s} + \gamma$ and $(K^-d)_{2p} \rightarrow (K^-d)_{1s} + \gamma$, induced by strong low-energy interactions, relative to the rates of the pure electric dipole transitions $2p \rightarrow 1s + \gamma$, are of order of one percent.

Precisions of this order $\pm 0.2\%$ and $\pm 1.0\%$ have been reached in the experiments of the PSI Collaboration [26] for the measurements of the energy level shift and width of the ground state of pionic hydrogen, respectively. These precisions are defined by the accuracy of the measurements of the X -ray spectra and yields in pionic hydrogen.

Measurements of the energy level displacements of the ground states for kaonic atoms at the same level of precision, would demand to take into account the rates of the transitions $(K^-p)_{np} \rightarrow (K^-p)_{1s} + \gamma$ and $(K^-d)_{np} \rightarrow (K^-d)_{1s} + \gamma$, induced by strong low-energy interactions, for the theoretical description of the experimental data on the X -ray spectra and yields in kaonic atoms.

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